MfP I, Woche 2, Seminar

20. Oktober 2022

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Behauptung Seien $f: X \to Y$ und $g: Y \to Z$ bel. Fkt.

$$(f \text{ surj. } \& \ g \text{ surj.}) \Rightarrow \underline{g \circ f} \text{ surj.}$$

$g \circ f : X \longrightarrow Z$

Beweis „zu zeigen"

$$Z: (g \circ f)(X) = Z$$

$$(g \circ f)(X) \overset{\text{Def}^{n}}{=} \{(g \circ f)(x) \mid x \in X\}$$

$$\overset{\text{Def}^{n}}{=} \{ g(f(x)) \mid x \in X\}$$

$$\overset{\text{Def}^{n}}{=} g(\{ f(x) \mid x \in X\})$$

$$\overset{\text{Def}^{n}}{=} g(f(X))$$

$$= g(Y) \text{ weil } f \text{ surj. ist}$$

$$= Z \text{ weil } g \text{ surj. ist} \qquad \square$$

es gilt:

$$f(X) = \{f(x) \mid x \in X\}$$

$$h(A) = \{h(a) \mid a \in A\}$$

$f\,;\,g$

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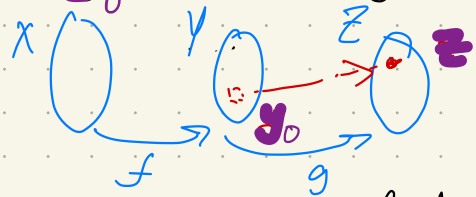
Beweis (Alternativ):

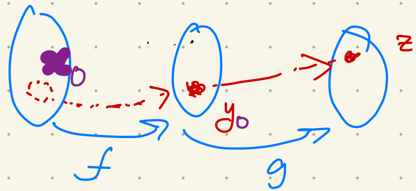
Sei $z \in Z$ bel. Z.z.: es existiert $x \in X$, so dass

$$(g \circ f)(x) = z$$

1) Wegen Surjektivität von g ex. ein $y_0 \in Y$ s.d. $g(y) = z$ $(\dagger)$



2) Wegen Surjektivität von f ex. ein $x_0 \in X$ s.d. $f(x) = y_0$ $(*)$



Daraus folgt $(g \circ f)(x_0) = g(f(x_0)) \overset{*}{=} g(y_0) \overset{\dagger}{=} z.$ $\square$

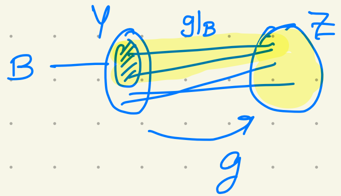
Seien $f, \tilde{f} : X \longrightarrow Y$ und $g : Y \longrightarrow Z$ Fkt. mit $\tilde{f}$ bijektiv. 2

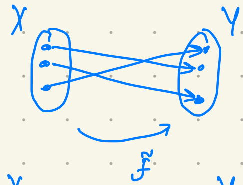
Sei auch $B \subseteq Y$.

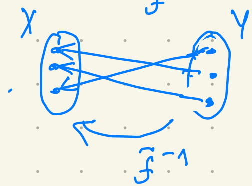
$\sim \mathcal{G}$, G

Zu „berechnen": $\mathrm{Gph}(g \circ f)$, $\mathrm{Gph}(g|_B)$, $\mathrm{Gph}(\tilde{f}^{-1})$

$$\mathrm{Gph}(f) \overset{\text{Def}^n}{=} \{ (x,y) \in X \times Y \mid f(x) = y \}$$







$$\mathrm{Gph}(g|_B) \overset{\text{Def}^n}{=} \{ (y,z) \in B \times Z \mid g|_B(y) = z \}$$

$$= \{ (y,z) \in B \times Z \mid g(y) = z \}$$

$$\left[\mathrm{Gph}(g) \; = \; \{ (y,z) \in Y \times Z \mid g(y) = z \} \right]$$

$$= \{ (y,z) \in B \times Z \mid (y,z) \in \mathrm{Gph}(g) \}$$

$$= B \times Z \cap \mathrm{Gph}(g)$$

$g|_B : B \longrightarrow Z$

$\forall y \in B : g|_B(y) = g(y)$

$\tilde{f}^{-1} : Y \longrightarrow X$

$\forall x \in X : \forall y \in Y : \; \tilde{f}(x) = y$

$\Longrightarrow \tilde{f}^{-1}(y) = x$

$$\mathrm{Gph}(\tilde{f}^{-1}) = \{ (y,x) \in Y \times X \mid \tilde{f}^{-1}(y) = x \}$$

$$= \{ (y,x) \in Y \times X \mid \tilde{f}(x) = y \}$$

$$= \{ (y,x) \in Y \times X \mid (x,y) \in \mathrm{Gph}(\tilde{f}) \}$$

$$= \mathrm{Gph}(\tilde{f})^{-1}$$

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$$\begin{aligned}
\mathrm{Gph}(g\circ f) &= \{(x,z)\in X\times Z \mid (g\circ f)(x)=z\} \qquad g\circ f: X \longrightarrow Z\\
&= \{(x,z)\in X\times Z \mid g(f(x))=z\}\\
&= \{(x,z)\in X\times Z \mid \exists\, y\in Y:\ f(x)=y \wedge g(y)=z\}\\
&= \{(x,z)\in X\times Z \mid \exists\, y\in Y:\ (x,y)\in \mathrm{Gph}(f)\\
&\qquad\qquad \wedge\ (y,z)\in \mathrm{Gph}(g)\ \}
\end{aligned}$$

$$\mathcal{P}(\emptyset) \overset{\text{Def}^{\text{n}}}{=} \{A \mid A\subseteq \emptyset\}$$

$$= \{\underset{1}{\emptyset}\} \neq \underset{0}{\{\ \}}$$

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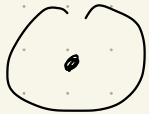

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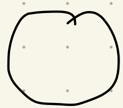

$$\mathcal{P}(\{\emptyset\}) = \{A \mid A\subseteq\{\emptyset\}\}$$

$$= \{\emptyset,\ \{\emptyset\}\}$$

$$\mathcal{P}(\{a,b\}) = \{\emptyset,\ \{a\},\ \{b\},\ \{a,b\}\}$$

$$\mathcal{P}(\{a,\emptyset\}) = \{\emptyset,\ \{a\},\ \{\emptyset\},\ \{a,\emptyset\}\}$$

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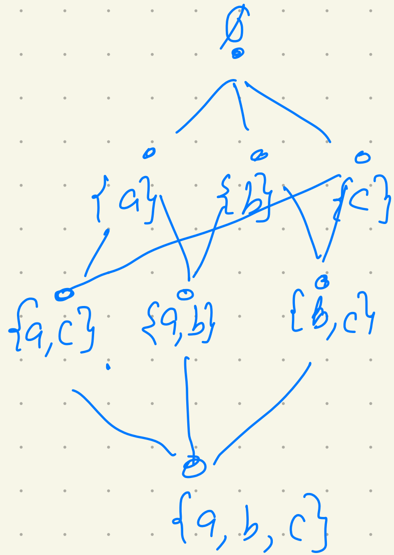

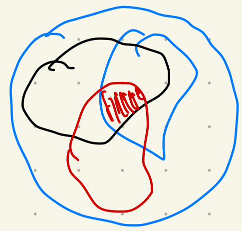

$$\bigcap_{p\in\mathbb{P}} \underbrace{\{n\in\mathbb{N} \mid \Phi(n,p)\}}_{A_p}$$

$$= \{n\in\mathbb{N} \mid \forall p\in\mathbb{P} : \Phi(n,p)\}$$

$$= \{n \mid n \text{ in } A_p \text{ für alle } p\in\mathbb{P}\}$$

$$\bigcap_{p\in\mathbb{P}} \{n \mid n\le 3 \text{ od. } n\ge p\}$$

$$= \{1,2,3\},$$